The Common Mathematical Foundation of ACG's ICM and AICM and the K&S PME

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INTRODUCTION

First, a note about acronyms in this research brief:

ACG, of course, refers to Alignment Capital Group. ICM refers to ACG's Index Comparison Method as implemented in QuanTrack, ACG's portfolio diagnostic and investment screening tool. AICM refers to ACG's Alternative Index Comparison Method, calculated in the same way as the ICM but with all private market invested capital provided by shorting (selling) the public market index. K&S PME refers to the public market equivalent calculation featured in the groundbreaking article by Steven Kaplan[1] and Antoinette Schoar[2] ("K&S") entitled "Private Equity Performance: Returns, Persistence and Capital Flows", The Journal of Finance, LX, 4 (August 2005), pp.1797-1823. This article was cited in the October 2007 edition of Private Equity Analyst as one of the five most important academic articles ever written on private equity.

Craig Nickels, my former partner,[3] and I invented the ICM calculation in late 1992 and early 1993. At first, we used the ICM only to manage the private equity investments of The University of Texas System, but beginning in 1994 we began to share it with other institutional investors, beginning with the Virginia Retirement System.

In about 1996, Venture Economics began to report the ICM return in its annual Yearbook of Private Equity, at first as the Long-Nickels-Coller (LNC) measure, then as the Bannock-Long-Nickels-Coller (BLNC) measure and finally as the public market equivalent (PME) return. Also in the late 90s, Craig and I worked with The Burgiss Group, Inc. to incorporate the ICM return into Private i, the industry-leading private equity portfolio management software package, which also referred to the calculation as the public market equivalent (PME) return. As a result of its adoption by both Venture Economics and The Burgiss Group, the ICM (a/k/a PME) calculation became a de facto industry standard for most major U.S. institutional investors as a measure of performance relative to any published public market index.[4]

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[1] Neubauer Family Professor of Entrepreneurship and Finance, Graduate School of Business, University of Chicago

[2] Michael M. Koerner Associate Professor of Entrepreneurial Finance at MIT's Sloan School of Management

[3] Craig Nickels is now Director of Private Equity at Washington University in St. Louis.

[4] For the original publication of the ICM (a/k/a PME) calculation, see "A Private Investment Benchmark," by Austin Long and Craig Nickels, a paper presented to the AIMR Conference on Venture Capital Investing in San Francisco on February 13, 1996.

As most readers will know, the ICM, as implemented in ACG's QuanTrack production software, calculates the opportunity cost of a private market investment by investing the same cash flows into a public market index as those invested in a private market investment and withdrawing the same cash from the public market index as those distributed by the same private market investment. The ICM uses the resulting terminal value of the index in order to calculate an index IRR that is in every way precisely comparable to the IRR of the private market investment. A private market IRR in excess of the ICM represents dollar-weighted, time-weighted private market performance in excess of the index (and, of course, vice versa).

Since the AICM shorts the index in order to fund the private market index, the terminal index valuation used to calculate the IRR of the index in the AICM is exactly equal to the terminal index valuation in the ICM, but with the signs reversed. Private market performance relative to the public index is determined in exactly the same way.

Kaplan and Schoar developed the K&S PME at the University of Chicago, apparently completely independent of any knowledge of ACG's ICM calculation and perhaps without any knowledge of (or at least without any express reference to) either the Venture Economics calculation or the Burgiss Group calculation. Briefly put, the K&S PME is the ratio between the future value of capital distributed by a private market investment (the numerator) and the future value of capital invested in a private market investment (the denominator), with both numerator and denominator compounded at rates determined by the performance of the index from the date of the cash flow to the horizon time period.

The purpose of this research brief is to demonstrate, using both closed form equations and numerical examples, the common mathematical foundation of ACG's ICM/AICM and the K&S PME in terms of the terminal value of the index. As the equations below will make clear, ACG's ICM and AICM both use the terminal value of the index in order to calculate the IRR of the index over the same time period and with the same cash flows as a private market investment – a true measure of opportunity cost. The K&S PME, on the other hand, uses the ratio between two index calculations (the future value of capital invested at the index return rate and the future value of capital distributed at the index return rate), the net of which is the terminal value of the index. Thus, as different as they may at first seem, the ACG ICM/AICM and the K&S PME have the terminal value of the index in common and each can therefore be translated into the other.

THE COMMON FOUNDATION OF THE ICM/AICM AND THE K&S PME

The K&S PME is calculated as follows:[5]

1. $\frac{FV_{Returned}}{FV_{Invested}} = PME_{K\&S}$ where $FV_x = \sum_{t=1}^{T} CF_t\left(1+i_t\right)^{T-t}$ and

$$i_t = \left(\frac{Index_T - Index_t}{Index_t}\right)^{1/T-t} - 1$$

In essence, PMEK&S is a times money earned calculation (TME, calculated as $*Returned* / $*Invested*) that uses the future values of the cash flows involved rather than their absolute values. Solving Equation 1 for $FV_{Returned}$, we obtain the K&S PME in terms of the future value of distributions from the private market investment:

2. $FV_{Returned} = FV_{Invested} PME_{K\&S}$

The Original Investment Comparison Method (ICM)

The ending value of the index used to calculate the ICM IRR, or opportunity cost, featured in QuanTrack, ACG's production software) is:

3. $FV_{Invested} - FV_{Returned} = Value_{Index_{Ending}}$

Note carefully that in the production ICM calculation, $FV_{Invested}$ is a long position and $FV_{Returned}$ is a short position. Thus, a successful private equity investment will result in a short position in $Value_{IndexEnding}$ and that short position will decrease the return of the index relative to the return of the private equity investment.

Solving Equation 3 for $FV_{Returned}$, we obtain:

4. $FV_{Returned} = FV_{Invested} - Value_{Index_{Ending}}$

Substituting Equation 4 into Equation 1, we obtain:

[5] This version uses the future value of each cash flow from its date to the horizon date at the time-weighted rate of return of the index over that time period. It is possible to calculate the K&S PME using the geometric mean return of the index from the first cash flow date to the horizon time period, but the result can be extremely distorted by disproportionately large cash flows in interim periods. This research brief therefore uses the more exact method, which reflects the sum of the future values of all the separate cash flows at the unique return rate calculated from the date of each cash flow to the horizon date.

5. $$\frac{FV_{Invested} - Value_{Index_{Ending}}}{FV_{Invested}} = PME_{K\&S}$$

Equation 4 can be simplified as:

6. $$1 - \frac{Value_{Index_{Ending}}}{FV_{Invested}} = PME_{K\&S}$$

Equation 6 thus enables the analyst to solve for PME in terms of the terminal value of the index as computed using ACG's ICM calculation/algorithm.

Viewed from the ACG perspective, it is also possible to solve for the terminal value of the index in terms of the PME by substituting Equation 2 into Equation 3 as follows:

7. $$FV_{Invested} - FV_{Invested} PME_{K\&S} = Value_{Index_{Ending}}$$

Equation 7 can be simplified as follows:

8. $$FV_{Invested}\left(1 - PME_{K\&S}\right) = Value_{Index_{Ending}}$$

The Alternative Investment Comparison Method (AICM)

While ACG's ICM calculation assumes that the capital invested into the index is a long position, the alternative index comparison method (AICM) assumes the opposite – that is, the cash used to invest in the private market investment results, not from a source external to both the private market investment and the index, but from a short position in (i.e., a sale of) the index. Expressed in the same terms, the AICM calculation of the ending value of the index (the ending value used to calculate the AICM) is as follows:

9. $$FV_{Returned} - FV_{Invested} = Value_{Index_{Ending}}$$

Note carefully that the $FV_{Invested}$ variable represents a short (negative) position in the index, while the capital returned by the index represents a long position. Equation 9 can be expressed as a function of the future value of the amount invested:

10. $$FV_{Returned} = Value_{Index_{Ending}} + FV_{Invested}$$

Substituting Equation 10 into Equation 1, we obtain:

11. $$\frac{Value_{Index_{Ending}} + FV_{Invested}}{FV_{Invested}} = PME_{K\&S}$$

Equation 11 can be simplified as follows:

12. $$\frac{Value_{Index_{Ending}}}{FV_{Invested}} + 1 = PME_{K\&S}$$

Thus, Equation 12 enables the analyst to solve for PME in terms of the ending value of the index as determined by ACG'S AICM calculation.

Viewing the analysis from the ACG perspective, we can solve for the ending value of the index as determined by ACG's AICM calculation in terms of PME.

Substituting Equation 2 into Equation 9, we obtain:

13. $FV_{Invested} PME_{K\&S} - FV_{Invested} = Value_{Index_{Ending}}$

Equation 13 can then be simplified as follows:

14. $FV_{Invested}\left(PME_{K\&S} - 1\right) = Value_{Index_{Ending}}$

In summary, Equations 6, 8, 12 and 14 demonstrate the intimate tie between the ICM and AICM calculation of terminal index value and the PME, each of which can be expressed in terms of the other.

NUMERICAL EXAMPLES

Figure 1 below calculates the 7.97% IRR of a private market investment beginning on August 31, 1980 and terminating on August 31, 2002.[6] It also calculates the 11.07% ICM IRR of the same cash flows invested into and distributed from the index with the same timing (the opportunity cost of the private market investment). Note that the private market investment underperformed the public market index by 310 basis points.

Private			Production Public		
					IRR > Index
IRR	7.97%			11.07%	-3.10%
Invested	$ (4,899,740)		Long position	$ (4,899,740)	
Returned	$ 8,043,880			$ 15,053,548	
TME	1.6			3.07	
	Original Long-Nickels Method				
	Public Market Long from Inception			Production Calculation	
	Private	S&P	Public Valuation Calc	Public IRR Calc	Value > Index
08/31/80	$ (2,000,000)	122	$ 2,000,000	($2,000,000)	$0
11/30/80	$ (154,857)	141	$ 2,451,311	($154,857)	$0
02/28/81	$ (281,015)	131	$ 2,570,964	($281,015)	$0
04/30/81	$ 25,175	133	$ 2,575,950	$25,175	$0
08/31/00	$ -	1518	$ 11,628,851	$0	$0
08/31/02	$ 4,747	916	$ 7,019,162	$7,014,415	($7,014,415)
Daily TWROR		0.02506%			
Annualized TWROR		9.58%			

Long position

Figure (1)

Figure 2 below calculates the same 7.97% IRR of the same private market investment but the opportunity cost calculation in this case is the AICM, which begins with a short position in (sale of) the index in order to fund the private market investment. Note that the AICM IRR is not calculable by the Excel XIRR function, primarily because the terminal value of the index is also short and the performance of the private market investment versus the index is therefore undefined.[7] Note also that the short position of the terminal value of the index is the exact dollar amount calculated in the ICM spreadsheet above but with the signs reversed.

[6] The blue dashed line in all of the spreadsheet numerical examples in this research brief represents the numerous lines omitted in order to make it possible to fit the spreadsheet onto the page.

[7] ACG has developed and is testing a method for determining IRR that is always defined. This new IRR calculation method will be subject of an upcoming ACG research brief.

Private		Alternative Public		
				IRR > Index
IRR	7.97%		#NUM!	#NUM!
Invested	$ (4,899,740)		$ (4,899,740)	
Returned	$ 8,043,880		$ 1,029,465	
TME	1.64		0.21	
	Modified Long-Nickels Method			
	Public Market Short at Inception			

Short position

	Private	S&P	Public Valuation Calc	Public IRR Calc	Value > Index
08/31/80	$ (2,000,000)	122	$ (2,000,000)	($2,000,000)	$0
11/30/80	$ (154,857)	141	$ (2,451,311)	($154,857)	$0
02/28/81	$ (281,015)	131	$ (2,570,964)	($281,015)	$0
04/30/81	$ 25,175	133	$ (2,575,950)	$25,175	$0
08/31/00	$ -	1518	$ (11,628,851)	$0	$0
08/31/02	$ 4,747	916	$ (7,019,162)	($7,014,415)	$7,023,909

Daily TWROR	0.02506%
Annualized TWROR	9.58%

Short position

Figure (2)

Figure 3 below calculates the K&S PME for the same cash flows underlying the two spreadsheets above. As the equations derived in the previous section predict, the net of the future values of the capital invested and the capital distributed is the exact dollar amount of the terminal position in the AICM calculation immediately above. The fractional K&S PME indicates that the private market index underperformed the index over its life.

Kaplan & Schoar PME Method **Using Multiple-Period TWRORs**				
Elapsed Days	Daily Total Return to Valuation Horizon	Future Value of Private Equity Invested Capital	Future Value of Private Equity Distributed Capital	K&S Net
8035	0.00025056	$14,970,910.28	$0.00	
7944	0.00023602	$1,009,536.26	$0.00	
7854	0.0002474	$1,961,071.53	$0.00	
7793	0.00024784	$0.00	$173,647.96	
730	-0.0006913	$0.00	$0.00	
0	0	$0.00	$4,747	
		$35,245,875.90	$28,231,461.04	($7,014,414.86)

K&S PME	0.8010

Figure (3)

Figure 4 below represents the ICM calculation for a private market investment over the same time period as the analyses above but with slightly different cash flows. In this case, the private market investment outperforms the index by 154 basis points and the terminal value of the index is therefore negative (i.e., short).

Private		ICM: Production Public		
				IRR > Index
IRR	32.66%		31.11%	1.54%
Invested	$ (3,054,597)		$ (3,054,597)	
Returned	$ 40,726,103		$ (27,033,781)	
TME	13.33		-8.85	
	Original Long-Nickels Method			
	Public Market Long from Inception		Production Calculation	

	Private	S&P	Public Valuation Calc	Public IRR Calc	Value > Index
08/31/80	$ (154,857)	122	$ 154,857	($154,857)	$0
11/30/80	$ (154,857)	141	$ 332,668	($154,857)	$0
02/28/81	$ (281,015)	131	$ 591,785	($281,015)	$0
04/30/81	$ 25,175	133	$ 573,553	$25,175	$0
08/31/00	$ 1,734,129	1518	$ (112,244,055)	$1,734,129	$0
08/31/02	$ 4,747	916	$ (67,750,390)	($67,755,137)	$67,755,137

Daily TWROR	0.02506%
Annualized TWROR	9.58%

Long position

Short position

Figure (4)

Note that the ICM (opportunity cost) IRR's slightly inferior return is the result of an enormous $67.8 million short position in the terminal value of the index. The reason for the apparent absurdity of such a large negative terminal value and the very slight difference in return it causes is that the present value of the negative terminal value at the IRR discount rate of 31.11% is only about $175,000. In other words, at very high discount rates, the terminal value of the index becomes very close to immaterial to the outcome of the IRR computation.

Figure 5 below uses the same cash flows but features the AICM calculation, which results in underperformance of the private market investment by 113 basis points and a positive terminal index valuation. Note that the dollar amount of the terminal index valuation is exactly the same as the ICM calculation immediately above but with the sign reversed. Note also that, in keeping with the analysis immediately above, the $67.8 million long position in the index creates only a marginally higher return in the index than in the private market investment to which it is compared.

Private		AICM: Alternative Public			
			Production Calculation		
					IRR > Index
IRR	32.66%	Short position		33.787%	-1.13%
Invested	$ (3,054,597)			$ (3,054,597)	
Returned	$ 40,726,103			$ 108,476,492	
TME	13.33			35.51	
	Modified Long-Nickels Method				
	Public Market Short at Inception				
	Private	S&P	Public Valuation Calc	Public IRR Calc	Value > Index
08/31/80	$ (154,857)	122	$ (154,857)	($154,857)	$0
11/30/80	$ (154,857)	141	$ (332,668)	($154,857)	$0
02/28/81	$ (281,015)	131	$ (591,785)	($281,015)	$0
04/30/81	$ 25,175	133	$ (573,553)	$25,175	$0
08/31/00	$ 1,734,129	1518	$ 112,244,055	$1,734,129	$0
08/31/02	$ 4,747	916	$ 67,750,390	$67,755,137	($67,745,643)
Daily TWROR		0.02506%			
Annualized TWROR		9.58%			

Long position

Figure (5)

Finally, Figure 6 below uses these same cash flows to calculate the K&S PME, which indicates that the private market investment strongly outperformed the public market index.

Kaplan & Schoar PME Method				
Using Multiple-Period TWRORs				
Elapsed Days	Daily Total Return to Valuation Horizon	Future Value of Private Equity Invested Capital	Future Value of Private Equity Distributed Capital	K&S Net
8035	0.00025056	$1,159,177	$0	
7944	0.00023602	$1,009,536	$0	
7854	0.0002474	$1,961,072	$0	
7793	0.00024784	$0	$173,648	
730	-0.0006913	$0	$1,046,719	
0	0	$0	$4,747	
		$21,434,142	$89,189,279	$67,755,137
			K&S PME	4.1611

Figure (6)

The table below demonstrates the relationship between the ordinary TME calculated by QuanTrack which of course involves no compounding, and the K&S PME, which compounds each cash flow at the index rate through the terminal valuation date:

	Uncompounded	Compounded	Compounded / Uncompounded		
Invested	$ 3,054,597	$ 21,434,142	7.0170		
Returned	$ 40,726,103	$ 89,189,279	2.1900		
TME (Returned / Invested)	13.3327	**4.1611**	9.2070		
			↓	Difference (Rounding)	Percentage Difference
Uncompounded TME - Compounded TME		9.1716	9.2070	-0.0353	0.3854%

Figure (7)

It is clear from this analysis that the capital invested component, which predominately occurs at dates well before the dates of the distributions from the investment, has much more of an effect on decreasing the PME multiple relative to TME than the capital returned component.

Alternatively, it is possible to analyze the relationship between TME and the K&S PME using an adaptation of the dimensional analysis of ACG's patented neutrally-weighted portfolio attribution analysis as follows:

	Timing	Return	Invested	Returned	Net	Multiple
I	Zero-Base	Zero	$ 3,054,597	$ 40,726,103	$ 37,671,506	13.3327
II *	Zero-Base	Actual	$ 22,865,049	$ 304,853,417	$ 281,988,368	13.3327
III **	Actual	Zero				
IV	Actual	Actual	$ 21,434,142	$ 89,189,279	$ 67,755,137	4.1611

I. Base TME	$ 3,054,597	$ 40,726,103	$ 37,671,506	13.3327
II - I Return Effect	$ 19,810,452	$ 264,127,314	$ 244,316,862	0.0000
IV - II Timing Effect	$ (1,430,907)	$ (215,664,138)	$ (214,233,231)	-9.1716
IV Actual Outcome	$ 21,434,142	$ 89,189,279	$ 67,755,137	**4.1611**

* Based on the TWROR from the first day of the investment to the index valuation date.

** Omitted, since with zero return actual timing equals zero-base timing.

Figure (8)

This second analysis again makes it clear that the primary cause of the decrease in PME relative to TME is the timing with which the cash flows occur, since the return effect is neutral in terms of the PME multiple.

CONCLUSION

As the equations and numerical examples above indicate, it is possible to convert ACG's ICM and/or AICM to the K&S PME and vice versa. ACG is currently evaluating client interest in introducing the K&S PME into QuanTrack.

The table below summarizes briefly the relationships between the ICM/AICM and the K&S PME:

Attribute	ICM	AICM	PME
Initial Position in the Index	Long	Short	Short
Ending Value of Index, PE > Index	Short	Long	Long
Ending Value of Index PE < Index	Long	Short	Short
Use of Ending Value of Index	IRR	IRR	None
Use of Components of Ending Value (FV of Invested and FV of Distributed)	None	None	Ratio (PME)

Alignment Capital Group is a full-service private equity consulting firm based in Austin, Texas. The firm's mission is to understand private equity as an asset class in a portfolio context, and thus to assist our clients in making optimal investment decisions.

Austin Long is the principal owner of Alignment Capital Group. His responsibilities include providing strategic portfolio management advice and conducting original research.

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